

# Preface

Quantum mechanics was already an old and solidly established subject when the first edition of this book appeared in 1966. The context in which a graduate text on quantum mechanics is studied today has changed a good deal, however. In 1966, most entering physics graduate students had a quite limited exposure to quantum mechanics in the form of wave mechanics. Today the standard undergraduate curriculum contains a large dose of elementary quantum mechanics, and often introduces the abstract formalism due to Dirac. Back then, the study of the foundations by theorists and experimenters was close to dormant, and very few courses spent any time whatever on this topic. At that very time, however, John Bell's famous theorem broke the ice, and there has been a great flowering ever since, especially in the laboratory thanks to the development of quantum optics, and more recently because of the interest in quantum computing. And back then, the Feynman path integral was seen by most as a very imaginative but rather useless formulation of quantum mechanics, whereas it now plays a large role in statistical physics and quantum field theory, especially in computational work.

For these and other reasons, this book is not just a revision of the 1966 edition. It has been rewritten throughout, is differently organized, and goes into greater depth on many topics that were in the old edition. It uses Dirac notation from the outset, pays considerable attention to the interpretation of quantum mechanics and to related experiments. Many topics that did not appear in the 1966 edition are treated: the path integral, semiclassical quantum mechanics, motion in a magnetic field, the  $S$  matrix and inelastic collisions, radiation and scattering of light, identical particle systems and the Dirac equation.

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Kurt Gottfried has here, at last, the opportunity to thank colleagues who have, either in writing or face-to-face, shared their knowledge, patiently answered questions and corrected misconception: in particular Aage Bohr, Michael Berry, Roy Glauber, Daniel Greenberger, Martin Gutzwiller, Eric Heller, Michael Horne, Ben Mottelson, Michael Nauenberg, Abner Shimony, Ole Ulfbeck and Anton Zeilinger, and the late John Bell, Rudolf Peierls and Donald Yennie. He is deeply indebted to David Mermin and Yuri Orlov for innumerable enlightening and stimulating discussions over many years.

We both wish to take this occasion to thank and remember those who first taught us quantum mechanics: David Jackson, Julian Schwinger and Victor Weisskopf.

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### Road Signs

This book treats considerably more material than fits into a standard *two-semester* course. Furthermore, the rigid organization of this (or any) book does not map in a simply-connected manner onto a good course. We therefore offer some guidance based on our experience with what we have found suitable for the *first* semester. Personal judgments and interests should be used to select material for the second semester, or for self-instruction. Various selections have been taught by us in the first semester. The emphasis has been on physical phenomena, with the general theory and approximation techniques intermingled with applications. The last time the semester was confined to general theory as motivated and illustrated by various one-body problems:

- The portions of the first three chapter with which students are not yet familiar, but leaving almost all of § 2.6 - § 2.9 on propagators, the path integral and semiclassical quantum mechanics for later.
- Low Dimensional Systems: spectroscopy in two-level systems, the harmonic oscillator and motion in a magnetic field.
- Hydrogenic atoms: fine and hyperfine structure, and the Zeeman and Stark effects, i.e., most of chapter 5, leaving aside Pauli's solution (§ 5.2).
- Symmetries: the rotation group (§ 7.4), some consequences of rotational symmetry (part of § 7.5); and tensor operators (§ 7.6).
- Elastic Scattering: general theory (§ 8.1 - § 8.2), approximation methods (§ 8.3), and scattering of particles with spin (§ 8.5).

*References.* Complete citations are given in Endnotes and the text, except for the following which recur often and are denoted by abbreviations:

Bethe and Salpeter: *Quantum Mechanics of One- and Two-Electron Atoms*, H.A.

Bethe and E.E. Salpeter, Springer-Verlag (1957)

Jackson: *Classical Electrodynamics*, J.D. Jackson, 3rd. ed., Wiley (1999)

LLQM: *Quantum Mechanics*, L.D. Landau and E.M. Lifshitz, 3rd. ed., Pergamon (1977)

WZ: *Quantum Theory and Measurement*, J.A. Wheeler and W.H. Zurek (eds.), Princeton (1982)

*Equation numbering.* Equations are numbered sequentially in each chapter, and those in different chapters are cited as  $(n.m)$ , where  $n$  is the number of the other chapter.

*Errata.* Corrections should be sent to both kg13@cornell.edu and ty18@cornell.edu. They will be posted on <http://www.lepp.cornell.edu/books/QM-I/>



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